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Some Considerations About the Frequency Dependence of the Characteristic Impedance of Uniform Microstrips

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Abstract—**Various possible definitions of characteristic impedance are derived from two different microstrip models obtaining similar results. It is shown that slightly different definitions yield strongly different behavior versus frequency.**

I. Introduction

THE FREQUENCY DEPENDENCE of the characteristic impedance $Z_0(\omega)$ of uniform microstrips has been discussed in various papers [4]–[10] with completely different results. In Denlinger's approach [4] $Z_0(\omega)$ is a decreasing function, while the other authors have obtained an increasing behavior.

In this paper it is shown how it is possible to obtain such different results, starting from the definitions of voltage, current, and power in microstrip.

Microstrips are transmission lines difficult to analyze. A microstrip, due to the presence of two distinct dielectric media, cannot carry a pure TEM mode. In most practical cases it can be assumed that only one mode (referred to as the fundamental mode) does propagate; however, the pertinent propagation constant γ does not depend linearly on the frequency; this is the dispersion phenomenon. This situation of single-mode propagation is assumed, in general, in the available theoretical calculations. Regarding measurements, γ or $\varepsilon_e = -\gamma^2/\omega^2\varepsilon_0\mu_0$ can be determined by means of techniques which eliminate the effects of the strip terminations, such as the use of sliding probes or by comparing lines of different lengths [11]. This may explain the availability of computations of γ which (with some adjustment of parameters) compare well with measurements [1], [2].

Insofar as the characteristic impedance is concerned, the case is far more involved. Strictly speaking, we could only define the characteristic impedance pertinent to each mode; then when one speaks of the characteristic impedance of the microstrip, it is intended, either explicitly or not, that some hypothesis is made about the mode coupling imposed by the terminations.

The various $Z_0(\omega)$ definable are different. In the present work we shall consider a number of different possible definitions of Z_0, on the basis of two microstrip models, due to W. J. Getsinger and H. J. Carlin, respectively.

II. Getsinger's Model

In this model [1] the microstrip fundamental mode is approximated by a longitudinal-section-electric mode. However, as the actual structure precludes a direct analysis, the microstrip (Fig. 1(a)) is substituted by the structure of Fig. 1(b); here the three unknowns (a', b', and u) are found by supposing that the new structure exhibits the same per-unit-length capacitance and inductance as the original one. A third equation arises when we suppose that the calculated γ fits exactly with a measured value at a given frequency, e.g., at 10 GHz. With this adjustment Getsinger was able to give a good analytical expression for γ. The analysis of the structure of Fig. 1(b) yields [1]:

$$H_{z,i} = -A_i\gamma\gamma_i sh(\gamma_i\psi_i) \quad (1)$$

$$H_{\psi,i} = -A_i\gamma^2 ch(\gamma_i\psi_i) \quad (2)$$

$$E_{\eta,i} = s\mu_0\gamma A_i ch(\gamma_i\psi_i) \quad (3)$$

$$\gamma^2 + \gamma_i^2 = s^2\varepsilon_0\varepsilon_i\mu_0 \quad (4)$$

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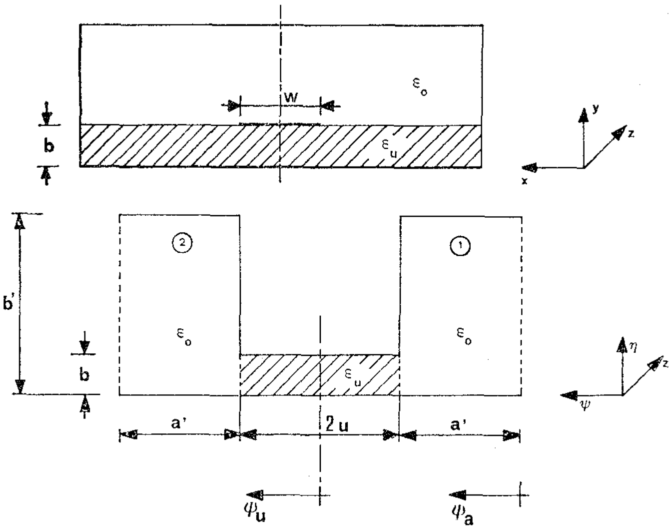


Fig. 1. (a) Cross-section view of a typical microstrip line. (b) Microstrip model according to Getsinger.

where $s = j\omega$, $i = u$ in the dielectric-filled region, $i = a$ in the air-filled region, $\varepsilon_a = 1$, ε_u is the relative permittivity of the dielectric, and γ_u and γ_a are suitable transverse propagation constants. By imposing proper boundary conditions on the air-dielectric discontinuities, (1)–(4) give $\gamma(\omega)$, as said above. In order to define the characteristic impedance, let us introduce the following quantities:

$$\bar{V}_a = -\int_0^{a'} \frac{b'}{a'} E_{\eta,a} \cdot d\psi_a$$

$$\bar{V}_u = -\int_0^{u} \frac{b}{u} E_{\eta,u}\, d\psi_u \tag{5}$$

$$\bar{V} = \frac{a'\bar{V}_a + u\bar{V}_u}{a' + u}. \tag{6}$$

These are the mean voltages across the air region, substrate region, and the whole section, respectively. In $\psi_u = 0$ we have the center voltage V_0

$$V_0 = -bE_{\eta,u}|_{\psi_u = 0}. \tag{7}$$

The total current is

$$I = 2\int_0^{a'} H_{\psi,a}\, d\psi_a + 2\int_0^{u} H_{\psi,u}\, d\psi_u \tag{8}$$

and, finally, the flowing power is

$$P = -2\int_0^{b'} d\eta \int_0^{a'} E_{\eta,a} H^*_{\psi,a}\, d\psi_a - 2\int_0^{b} d\eta \int_0^{u} E_{\eta,u} H^*_{\psi,u}\, d\psi_u. \tag{9}$$

Now we define the following characteristic impedances:

$$Z_1 = \frac{\bar{V}}{I};\ Z_2 = \frac{V_0}{I};\ Z_3 = \frac{P}{II^*}$$

$$Z_4 = \frac{V_0 V_0^*}{P};\ Z_5 = \frac{\bar{V}\bar{V}^*}{P}. \tag{10}$$

These expressions will be discussed later.

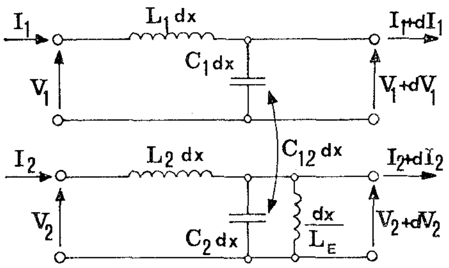


Fig. 2. Microstrip model according to Carlin.

III. Carlin's Model

In this model the microstrip is represented as the result of two uniform lines coupled through distributed capacitance [2] (Fig. 2). The modes have propagation constants given by

$$\gamma_1^2, \gamma_2^2 = \tfrac{1}{2}\{\gamma_{TEM}^2 + \gamma_{TE}^2 \mp \sqrt{(\gamma_{TEM}^2 - \gamma_{TE}^2)^2 + 4\gamma_{TEM}^2 K_1^2 K_2}\} \tag{11}$$

where

$$\gamma_{TEM} = s\sqrt{L_1 C_1}$$

$$\gamma_{TE} = \sqrt{s^2 L_2 C_2 + \frac{L_2}{L_E}} \tag{12}$$

$$K_1 = \frac{C_{12}}{C_1}$$

$$K_2 = \frac{L_2}{L_1}. \tag{13}$$

The minus sign in (11) gives γ_1, the plus sign gives γ_2. We assume that γ_2 is real (i.e., the higher mode is cut off), and $\gamma_1 = \gamma$ is imaginary and is pertinent to the fundamental mode. Carlin's model that gives an expression of γ in good agreement with measurements [11], is now used for impedance calculations. We shall consider a semi-infinite strip; then in $z = 0$ we have two ports, related to modes 1 and 2, respectively. Let V_1, I_1, V_2, I_2 be the respective voltages and currents. Now, in order to compare the results of Carlin's model to those of single-mode models, we shall assume that the microstrip is terminated (in $z = 0$) in such a way as to annihilate mode 2 in the microstrip ($z > 0$).

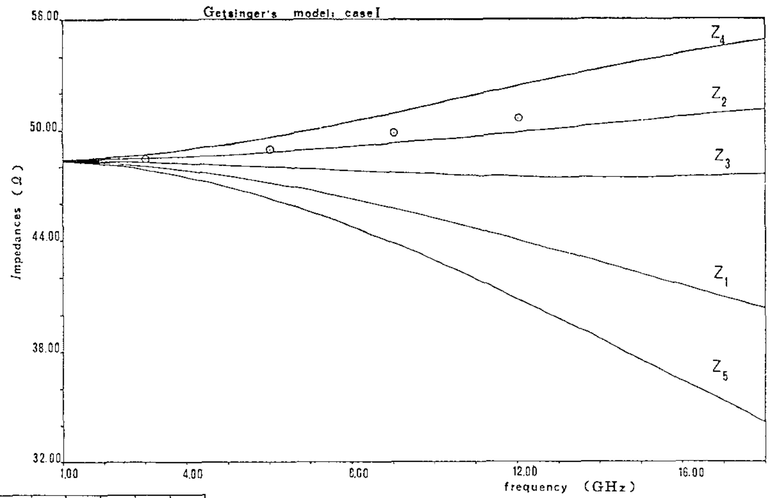
In order to give expressions analogous to (10) we note first that in $z = 0$

1) the total current I in the cross section is equal to I_1 since higher modes have zero-average magnetic field distribution,
2) similarly the mean voltage is $\bar{V} = V_1$,
3) the center voltage is now $V_0 = V_1 + V_2$, and
4) $P = V_1 I_1^* + V_2 I_2^*$ for we do not consider other modes.

Then we can define also for Carlin's model the same characteristic impedances given in (10).

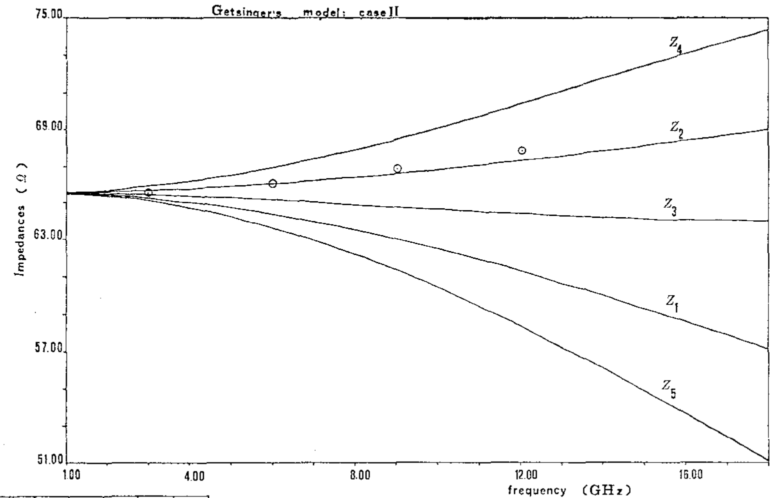
IV. Discussion

The expressions (10) for the characteristic impedances have been obtained, but the calculations are too lengthy and



	Z_1	Z_2	Z_3	Z_4	Z_5
Limit value (Ω) for f → 0	48.35	48.35	48.35	48.35	48.35
Limit value (Ω) for f → ∞	22.66	61.48	61.48	61.48	8.35

(a)



	Z_1	Z_2	Z_3	Z_4	Z_5
Limit value (Ω) for f → 0	65.52	65.52	65.52	65.52	65.52
Limit value (Ω) for f → ∞	28.96	86.42	86.42	86.42	9.70

(b)

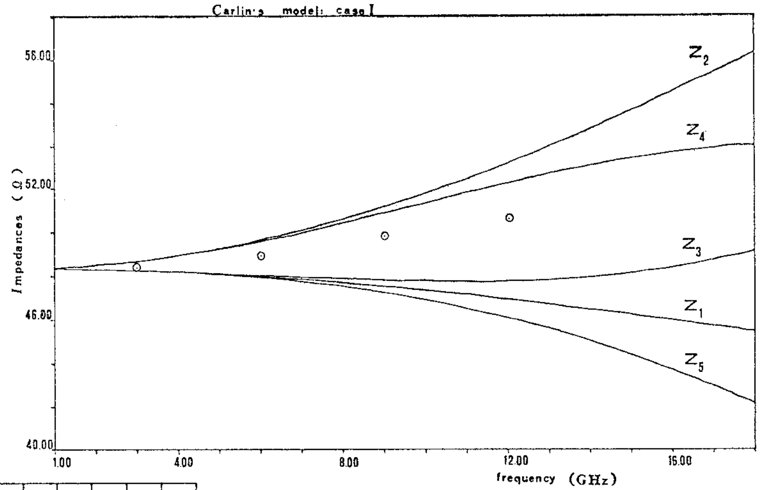
Fig. 3(a) and (b). Plot of the various characteristic impedance definitions according to Getsinger model. Case I: $b = 0.635$ mm; $w/b = 1$; $\varepsilon_u = 10$. Case II: $b = 0.635$ mm; $w/b = 0.5$; $\varepsilon_u = 10$. Circles represent results obtained by Krage and Haddad.

the final expression too complicated to report here. The reader shall find a complete analysis in [3]. Here we report the main results regarding the frequency behavior of $Z_1 + Z_5$, that is

1) at $\omega = 0$ all Z_k's are equal to the dc impedance $(LC)^{1/2}$, L and C being the per-meter inductance and capacitance,
2) Z_1 and Z_5 decrease for increasing frequency,
3) Z_2,Z_3,Z_4 increase for increasing frequency, and
4) the above is true for both models though the expressions of the individual Z_k's calculated with the two models are different.

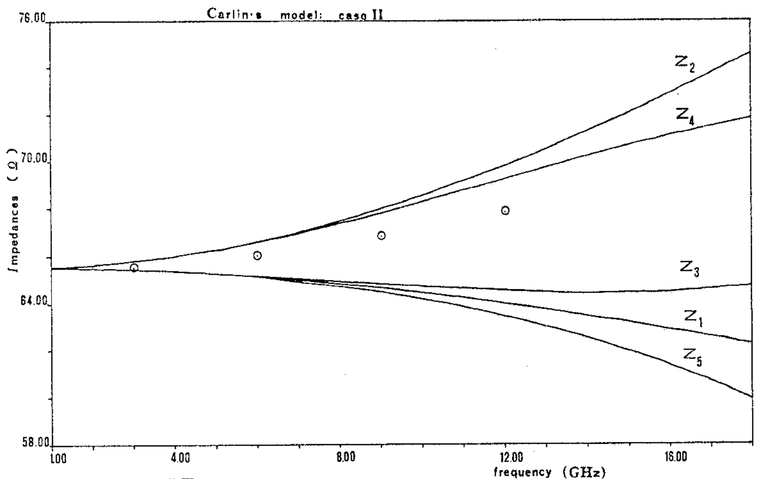
In Figs. 3 and 4 one can see the results of the various characteristic impedance definitions, using the Getsinger and Carlin models, respectively, compared with data obtained by field analysis through computer [8].

Summing up, we have considered a number of reasonable definitions of characteristic impedance, based on simplified models and have obtained different frequency behaviors. In particular, the definition $Z_1 = Z_1(0)\sqrt{\varepsilon_e(0)/\varepsilon_e(f)}$ is coincident with Denlinger's. The definitions Z_2 and Z_3 are analogous to those given in [5]–[10]. The frequency behaviors found by all these authors agree with ours. In these results, it must be stressed that slightly different definitions as, say, those of Z_1 and Z_2, yield strongly different behaviors; Z_1 decreases while Z_2 increases for increasing frequency. Though characteristic impedance is a useful concept



	Z_1	Z_2	Z_3	Z_4	Z_5
Limit value (Ω) for $f \to 0$	48.35	48.35	48.35	48.35	48.35
Limit value (Ω) for $f \to \infty$	40.06	80.00	61.48	104.41	26.10

(a)



	Z_1	Z_2	Z_3	Z_4	Z_5
Limit value (Ω) for $f \to 0$	65.52	65.52	65.52	65.52	65.52
Limit value (Ω) for $f \to \infty$	52.83	105.67	66.42	129.21	32.30

(b)

Fig. 4(a) and (b). Plot of the various characteristic impedance definitions according to Carlin model. Case I: $b = 0.635$ mm; $w/b = 1$; $\varepsilon_u = 10$. Case II: $b = 0.635$ mm; $w/b = 0.5$; $\varepsilon_u = 10$. Circles represent results obtained by Krage and Haddad.

in microstrip study, the above results show that particular care has to be taken both in computer calculations and in interpretation of experimental data to avoid confusion. In particular, it is hoped that the results sketched may be useful in the interpretation of computer-calculated results.

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